LEBANESE AMERICAN UNIVERSITY
Department of Computer Science and Mathematics

Discreet Structures I

Exam 1

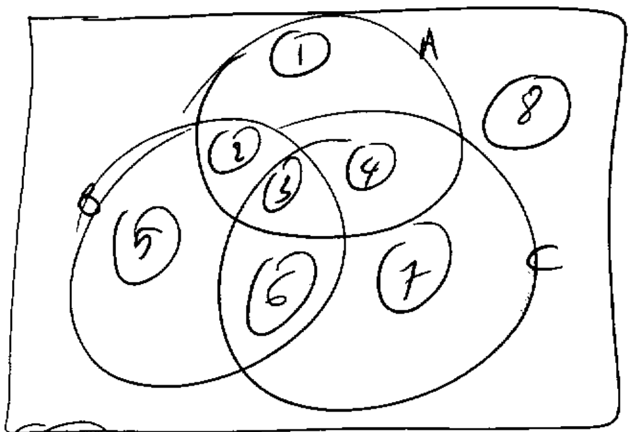
Fall 2013 (October, 2013)

Name: Solution **ID:**

Question Number	**Grade**
1. 9%	
2. 12%	
3. 8%	
4. 16%	
5. 14%	
6. 8%	
7. 7%	
8. 14%	
9. 12%	
Total	

9%

1. Show that $A-(B\cup C)=(A-B)\cap(A-C)$.



$A-(B\cup C)=$ 1

$A-B=$ 1, 4

$A-C=$ 1, 2

$\cap =$ 1

$\therefore =$

12%

2. If A,B and C are countable sets, show that $(A\times B)\cup C$ is also countable.

Spx $A=\{a_1, a_2, \quad , a_n, \dots\}$

$B=\{b_1, b_2, \dots, b_n, \dots\}$

$C=\{c_1, c_2, \dots, c_n, \dots\}$

$A\times B$ ~~is~~ is countable if we use

	a_1	a_2	.
b_1	(a_1,b_1)	(a_2,b_1)	
b_2	(a_1,b_2)	(a_2,b_2)	
|			
|			

by Cantor diagonalization.

$\therefore$ $A\times B$ can be seen as $=\{d_1, d_2, \dots, d_n, \dots\}$ since countable.

Now $(A\times B)\cup C=\{d_1, d_2, \dots, d_n, \dots\}\cup\{c_1, c_2, \dots, c_n, \dots\}$

can be read as : $\{d_1, c_1, d_2, c_2, d_3, c_3, \dots\}$

$\Longrightarrow$ Countable also.

3. Is it always true that if two sets A and B are such that $A \subset B$, then it should follow that $|A| < |B|$?

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No, if $A = \mathbb{N}$, $B = \mathbb{Z}$ $\Rightarrow$ $|A| = |B|$.

Applies to all countable subsets of countable sets !!

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4. Consider the proposition: **If you understand the material, you will pass this test.**

(a) Write down its *contrapositive.*

If you don't pass the test $\Rightarrow$ you don't understand the material

(b) Write down its *converse*

If you pass the test $\Rightarrow$ you understand the material.

(c) Specify the sufficient and necessary conditions in the statement?

p: you understand the material. SUFFICIENT cond

q: you pass the test : NECESSARY Cond

(d) Complete the equivalent statement to the one above: Only if .... then ....

Only if you pass, then you understand.

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5. If $x, y \in$ the set of all positive integers, and if $P(x, y)$ stands for y is a multiple of x then write an English translation for:

(a) $\forall x, \exists y, \sim P(x, y)$ There is an integer y that is not the multiple of any integer x.

(b) $\exists x, \forall y, P(x, y)$ There is an integer x that is a divisor of all integers y.

$\boxed{\text{or}}$

Any integer y is a multiple of some integer x

6. Write the negation of the statement: "If the weather permits, then if my friends are available tomorrow, we will party outdoors."

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The, weather permits and my friends are avaible yet we will not party outdoors.

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7. If you know that the proposition $(p \wedge q) \to r$ is false, what can you say about the truth value of the proposition $(p \vee \sim r) \to q \vee p$?

$p \wedge q \longrightarrow r : F \Rightarrow p \wedge q : T \; ; \; r : F$

$\Rightarrow (p, q, r) = (T, T, F) \Rightarrow$

$p \vee \sim r : T$ $\quad \boxed{q \vee p : T} \Rightarrow \boxed{p \vee \sim r \to q \vee p} : T$

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8. Show that the 2 statements: $\underbrace{(p \to r) \vee (q \to r)}_{\mathrm{I}}$ and $\underbrace{(p \wedge q)}_{\mathrm{II}} \to r$ are equivalent.(**Do not use truth tables**)

I : F if $p \to r : F$ $\quad q \to r : F \Rightarrow$

$p : T \; ; \; r : F, \quad q : T, \quad r : F \Rightarrow$

$\boxed{(p, q, r) = (T, T, F)}$

for I to be F.

II : $p \wedge q \to r : F \Rightarrow p \wedge q : T \; . \; r : F$

$\Rightarrow p, q : T, \; r : F \Rightarrow \boxed{(p, q, r) = (T, T, F)}$

$\Rightarrow$ I and II false in same situation

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9. Translate into symbols **in 2 different ways** using predicates and quantifiers **of your own choice**:"If you are a student in this class then you are a math major."

I

$x : \in$ student @ LAU.

$P(x)$: x is in this class.

$Q(x)$: x is a Math Major.

$\forall x : P(x) \longrightarrow Q(x)$.

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or $\quad x \in$ student in this class

II

$R(x)$: x is a Math major

$\forall x : R(x)$